THE UNIVERSITY OF CHICAGO

TIME'S UP
AN ANALYSIS OF TWO STRATEGIC DECISIONS IN BASKETBALL

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Abstract

This paper examines two strategic decisions made in the end of a basketball quarter and game: going two-for-one and intentionally fouling when up three. The paper attempts to build two simple models to analyze these situations. The data shows that going two-for-one is the correct decision. Going two-for-one is worth about .5 more net points than not going two-for-one. The paper also shows that intentionally fouling when up three might be the correct strategic decision, but cannot conclude with certainty. In the data set there are only 8 times I can identify when teams intentionally foul as opposed to 600 times when they do not foul. As such I can make no conclusions about fouls. In that small sample, teams that fouled never lost. The paper uses a unique event-by-event database from STATS Inc. The paper begins by briefly summarizing a history of sports analytics. A relevant literature review follows. [1]

[1] I want to thank my adviser Allen Sanderson. I also want to thank Grace Tsiang, Victor Lima, Alex Mackay, Kris Hult, Karen Hsu, Tova Levin, Jason Kass, Margaret Cantara, Louis Solmonson, Alex Speiser and Sam Jacobson for their advice. The data for this paper could not have been obtained without John Huizinga. My parents, Peninah Petruck and Arthur Jacobson have been invaluable as editors and supporters of my decision to write about basketball. I dedicate this paper to the memory of my uncle Richard Jacobson, who took me to countless Knicks-Bulls game. Lastly, I must thank the NY Knicks for inspiring my love of basketball.

Contents

1 Introduction

> "I am a big sports fan, baseball and basketball, everything. People will say to me, 'Does it really matter if the Knicks beat the Celtics?' And I think to myself, 'Well, it's just as important as human existence.' "
>
> -Woody Allen[2]

Sabermetrics is the study of baseball using statistics. From its creation, baseball fans have kept, analyzed, and debated statistics. By the middle of the first decade of the 2000s the *Moneyball* revolution occurred. Featured character Bill James' then-seemingly esoteric metrics such as on-base-percentage (OBP) and wins-above-replacement (WAR) were understood and demanded by the public.[3] Eventually hired as a consultant by the Red Sox, James confirmed Sabermetrics' relevance by helping them win their first World Series in 86 years. But baseball always lent itself to statistics, even before *Moneyball* made their use publicly popular. Basketball is just recently beginning to embrace advanced statistical analysis, as is the National Football League (NFL).

Compared with baseball, basketball is more difficult to measure and analyze. Baseball is a series of discrete events. Basketball is not. The game flows, one play to another. One seemingly horrid shot is offensively rebounded for an easy layup. Was the shot bad? Hard to say.[4] A driving layup on offense might allow the other team to score on a fast-break opportunity. One seemingly good action can lead to a an equally bad one. Basketball is also much more a of a team game than is baseball. Every player on the court has an influence on every play. When the game is over, one team wins. The complicated dynamics of how that game was won are not captured in the box score, and certainly not captured by the mainstream media's interpretation of that box score.

The modern box score tells us who won and who lost. However, it fails to tell us how we got there. It tells us how many points players scored and how many shots they took to get there. But not all shots are created equal. Not all rebounds or assists are equal. The time a player spends on the court matters only in how he affected the final score, not by what traditional statistics he

[2] Dodes, Rachel. Older, Mellower, but Still Woody. Wall Street Journal, June 15, 2012.

[3] Lewis, Michael. Moneyball. W. W. Norton & Company, 2011.

[4] Goldsberry shows how some missed shots by certain players (Kobe Bryant) are offensively rebounded for easy layups more often than others
The Kobe Assist. Grantland.com. http://www.grantland.com/story/_/id/8719297/how-kobe-bryant-missed-shots-translate-new-nba-statistic-kobe-assist.

logs. Advance statistical analysis tells basketball's story much better. APBRmetrics [5] attempts to use more advanced analytical techniques to analyze basketball. The name derives from a basketball discussion board: http://apbr.org/metrics/ .[6] The site became a home for basketball analytic nerds.[7]

The history of basketball analytics changed in the 1990s when Dean Oliver - a former player, coach, and statistics PhD - and John Hollinger - an economics major and journalist- began making advanced statistics more mainstream through online writing. In the early 2000s, both Hollinger and Oliver published books that to this day frame any discussion of basketball analytics. Oliver's *Basketball on Paper* (2003), discussed later in the paper, highlights the importance of measuring basketball statistics in the framework of possessions. Hollinger's *Pro-Basketball Prospectus* created a player ranking system called PER, which now, despite its many critics, dominates ESPN and is a starting point for most discussions of player evaluation. By 2004 Oliver was hired full-time by the Seattle Supersonics, and Hollinger became one of ESPN.com's main writers.[8]

The MIT Sloan Sports Analytics Conference was started by current Houston Rockets General Manager Darryl Morey in 2006.[9] Just a few MIT classrooms were used in the first years of the conference, but now– in 2013– it occupied the Boston Convention and Exhibition Center, with almost 2000 attendees. In 2013, 29 of 30 NBA teams had representatives at the conference.[10] Sports analytics are becoming mainstream.

Along with the growth of the MIT Conference, the use of statistical analysis has spread from baseball to other sports. Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers."[11] Any rigorous analysis of sports falls ino this field. Many papers now employ game theoretic models and analyses to make conclusions about in-game strategic decisions in sports. David Romer, an economist at University of California, Berkeley, used play-by-play data from the NFL to examine what teams should

[5] Association for Professional Basketball Research Metrics, is where this name comes from, it is the brother of Sabermetrics

[6] Originally http://www.sonicscentral.com/apbrmetrics/

[7] The site is still active, though less so than in the mid 2000s- many of the posters have been hired by NBA teams.

[8] About the Author. http://www.basketballonpaper.com/author.html.
John Hollinger, MIT Sloan Sports Analytics Conference. http://www.sloansportsconference.com/?p=414.

[9] http://apbr.org/metrics/

[10] Pelton: What I Learned at Sloan. ESPN.com. http://insider.espn.go.com/nba/story/_/page/PerDiem-130304/nba-key-points-mit-sloan-sports-analytics-conference.

[11] Myerson, Roger B. Game Theory: Analysis of Conflict. Harvard University Press, 1997. pp. vii-xi.

do on fourth down in football: kick or go for first down? He found large deviations from what teams should do to maximize winning. Teams should be going for it more often than they are. He quotes Milton Friedman: "unless the behavior of businessmen in some way or other approximated behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long...The process of natural selection thus helps to validate the hypothesis [of return-maximization]" [12] There then must be reasons for a deviation from what is certainly an optimal strategy. While my paper attempts to discover what those optimal strategies are for a few situations in basketball, I cannot fully explain why teams might deviate from those strategies.

Chicago economist Steven Levitt, along with Tim Groseclose, a political scientist at UCLA, and Pierre-Andre Chiappori, an economist at Columbia University, tested game theoretic predictions against empirical data on penalty kicks in soccer games.[13] They found that, for the most part, players are choosing the best strategy.

The literature on strategic decisions in baseball is vast. Papers covering nearly every strategic decision have been written, such as when to steal or bunt. Baseball decisions are not limited to on-field strategies; they also include personnel decisions.[14] The idea of tradeoffs, which I discuss in more detail later, is important for baseball. When a team decides to bunt, it is essentially trading an out for a better chance of scoring a run. Nate Silver, who actually started his analytical career not in politics but in baseball, calculated the costs and benefits of every out and hit for every situation in a baseball game.[15] He finds that, usually, bunting is more costly than beneficial.

Expanding on the sports literature, my paper seeks to expand on and provide backing for some conventional strategic decisions that coaches make in a typical National Basketball Association (NBA) game. I use detailed play-by-play data from four NBA seasons, collected and organized by STATS inc., to look at two specific on-court situations.[16]

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[12] Romer, David. Do Firms Maximize? Evidence from Professional Football. Journal of Political Economy 114, no. 2 (2006): 340365.

[13] Chiappori, P.-A., S. Levitt, and T. Groseclose. 2002. "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer ." The American Economic Review, 92(4): 1138-1151.

[14] Silver, Nate. Baseball Prospectus — Lies, Damned Lies: The PECOTA Top 100. Baseball Prospectus. http://www.baseballprospectus.com/article.php?articleid=6153.

[15] Baseball Prospectus — Lies, Damned Lies: Introducing ORVY. Baseball Prospectus. http://www.baseballprospectus.com/article.php?articleid=4003.

[16] I am using only data for the NBA, and thus can only make conclusions about basketball decisions in the NBA. But the same general objectives and constraints (with minor changes) should hold for other levels of basketball.

First, toward the end of a quarter in a basketball game, coaches will often tell their players to go "two-for-one." What does this mean? The coach will instruct his team when they get the ball with less than 48 seconds left, but more than 24 (shot clock). He will instruct his players to take a quick shot in order to guarantee getting the ball back in time for a second shot before the end of the quarter. So they are essentially trading two quick shots for one more developed shot. An example: Team A gets the ball with 40 seconds left in the quarter. If they take 20 seconds to get a shot off, Team B would then take the remaining 20 seconds, and Team A would only have a chance to score once. But if they go two-for-one they would take about 10 seconds and rush a shot. Team B would then get the ball back with 30 seconds. Finally, Team A would get the ball back (assuming no offensive rebound) with 6 seconds and enough time to get off another rushed shot.

The question I attempt to answer: Is going two-for-one worth it? Are those two quick shots better than a more developed one? I build a simple model to describe going two-for-one. I then refer to the play-by-play data for a few NBA seasons, and determine whether going two-for-one is the optimal strategy.

A second basketball decision I analyze involves whether to foul an opponent on purpose under certain circumstances. Up three points with less than 24 seconds to go, a team on defense will be faced with a decision: to foul or not. Why would a team foul? If they foul and send the offensive team to the line, the most points they can score (as long as it is not an "and-one") is the two free throws. If the defensive team did not foul, the offensive could hit a three pointer and tie the game. The tradeoff is: allowing the other team the chance to get two points to prevent them from the chance of getting three. I refer to the the play-by-play data to plug in actual numbers for Annis' model.

With 10 seconds to go in Game 4 of the NBA finals in 2009, Orlando Magic Coach Stan Van Gundy decided not to foul with his team up three points. After a Los Angeles Lakers timeout, Lakers' guard Derek Fisher hit a three-pointer with four seconds to force overtime LA won in overtime to take a 3-1 series lead. [17] The Lakers won the championship in the next game. Should Van Gundy have instructed his Magic to foul here? I will try and answer that question.

Decisions that can change a game by one or two points matter. The average NBA point

[17] http://scores.espn.go.com/nba/playbyplay?gameId=290611019

differential in the 2013 regular season between the last team in the playoffs, Milwaukee Bucks, and the first team out of the playoffs, the Philadelphia 76ers, was -1.5 for the Bucks and -3.3 for the 76ers.[18] 1.8 net point per game difference separated these two teams from making the playoffs.

In Section II, I review relevant and important basketball literature that has helped frame the basketball analytics discussion. In Section III, I build models of the decisions I am using. Section IV uses the play-by-play data to analyze the strategic decisions. Section V concludes.

Appendix A poses other questions that my data could be used to answer. Appendix B includes Stata code and robustness of my data. References follow Appendix B.

[18]http://espn.go.com/nba/standings

2 Literature Review

Dean Oliver (2004)[19] tried to rewrite traditional basketball statistics so that we can analyze performance on a basketball court. In baseball, we measure success over number of at-bats. In basketball, we have possessions in the denominator. Why possessions? These are the chances each team has to score. In every possession there is an outcome: a made basket, a turnover. For two teams, they have roughly (within in two) the same amount of possessions per game. Possessions are not included in the box score, so Oliver devised a formula to calculate them:

$$POSS_t = (FGM_t + \lambda FTM_t) + \alpha[(FGA_t - FGM_t) + \lambda(FTA_t - FTM_t) - OREB_t] +$$

$$(1-\alpha)DREB_O + TO_t \tag{1}$$

He tested this formula against the data and found it to be extremely accurate ($R^2 = 0.9473$) when using the below (reduced form) plugins for the variables:

$$POSS_t = 0.976 * (FGA_t + 0.44 * FTA_t - OREB_t + TO_t) \tag{2}$$

Oliver constructed a number of other important metrics that use possessions in the denominator. With possessions in the denominator we can evaluate how a team performs on offense and defense. These are called Offensive Rating ($ORtg_t$) and Defensive Rating ($DRtg_t$) (normalized to 100 possessions per game).

$$ORtg_t = \frac{PTS_t}{POSS_t} * 100 \qquad DRtg_t = \frac{PTS_O}{POSS_O} * 100 \tag{3}$$

We can see that these metrics will be helpful when evaluating the counterfactual of a play i.e. what is the chance a team would have scored on a given play if they had not turned over the ball; or what is the probability that, even though a team stole the ball, the offensive team would not have scored anyway (rendering the block or steal irrelevant). The net difference between the

[19] Oliver, Dean. Basketball on Paper: Rules and Tools for Performance Analysis. Potomac Books Inc., 2004.

two is called the net efficiency rating. Oliver concluded that there is little correlation between the two:

> Historically and in 2005- 06, there is very little correlation between offensive and defensive ratings; good offensive teams do not tend to be better or worse on defense.[20]

Using possessions, Oliver was able to create an adjustment to any typical box score for pace. A team that has double the possessions of another team gives its team double the chances to record box score statistics. Faster or slower pace is not necessarily good or bad, but it useful to be able to compare any in-game statistics from team to team. So Oliver described a pace adjustment. The metric below is for points per game, but we could apply this to any measured statistic. [21]

$$adjPTS = PTS * \frac{POSS_l}{POSS_t} \implies adjX = X * \frac{POSS_l}{POSS_t} \tag{4}$$

Shooting metrics are related to field goal percentage (FG%). In the traditional box score, FG% measures made field goals over total field goals attempted. But this metric ignores the added value of three-point field goals and free-throws. Oliver shows two more effective field goal percentage (eFG%) and true shooting percentage (TS%):

$$FG\% = \frac{FGM}{FGA} \qquad eFG\% = \frac{FGM + 0.5 * 3PM}{FGA} \qquad TS\% = \frac{\frac{PTS}{2}}{FGA + 0.44 * FTA} \tag{5}$$

eFG% accounts for made three pointers and TS% accounts for both three pointers and free throws. Oliver says:

> True shooting percentage provides a measure of total efficiency in scoring attempts, while effective field goal percentage isolates a players (or teams) shooting efficiency from the field. Both measures are appropriate for different situations.[22]

When looking at rebounds, pace can affect how many a player or team gets per game. What we really care about is what share of available rebounds did a player or his team collect.

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[20] *ibid*

[21] $POSS_l$ is league possessions

[22] *ibid*

Oliver gives us a rebound rate (or rebound percentage) for players and teams, for total rebounds (REB%), offensive rebounds (OREB%) and defensive rebounds (DREB%): Players:

$$REB\%_p = \left[\frac{REB_p}{REB_t + REB_O}\right]\left[\frac{MIN_t}{MIN_p}\right] \tag{6}$$

$$OREB\%_p = \left[\frac{OREB_p}{OREB_t + DREB_O}\right]\left[\frac{MIN_t}{MIN_p}\right] \tag{7}$$

$$DREB\%_p = \left[\frac{DREB_p}{DREB_t + OREB_O}\right]\left[\frac{MIN_t}{MIN_p}\right] \tag{8}$$

Team:

$$OREB\%_t = \left[\frac{OREB_t}{OREB_t + DREB_O}\right] \tag{9}$$

$$DREB\%_t = \left[\frac{DREB_t}{DREB_t + OREB_O}\right] \tag{10}$$

$$REB\%_t = \left[\frac{OREB\%_t + DREB\%_t}{2}\right] \tag{11}$$

Recognizing the influence and importance of Oliver and other analytical metrics, ESPN now regularly includes plus/minus in its box scores. Plus/minus measures the team point differential while a player is on the court (offensive points minus defensive points). There are two other types of plus/minus. Net plus/minus measures the plus/minus for a player while he is in the game as compared to when that player is not in the game. Adjusted plus/minus tries to account for the difference in teammates and opponents; it is a complicated formula that is described by Rosenbaum (2004).[23]

Linear weights are a weighted sum of different box score statistics. There exist many variations online, whether from the NBA, online bloggers, or columnists. John Hollinger (2004) de-

[23] Rosenbaum, Dan T. Measuring How NBA Players Help their Teams Win. 2004. 82games.com, http://www.82games.com/comm30.htm.

veloped *Player Efficiency Rating* (PER) which is likely the most famous linear weight model.[24] However there are many problems with linear weight models, as described by Oliver 2007:

> linear weights have numerous faults, including the frequently subjective weights applied to the statistics, the lack of defensive statistics available for such systems, the lack of correlation with winning at the team level, the theoretical difficulty in incorporating new statistics that may be developed, and the general lack of a measuring stick to calibrate their accuracy.[25]

In most NBA games, when a player gets one more foul than the number quarter it is, his coach will remove him until the end of the quarter. For example, if a player receives two fouls in the first quarter, he will be on the bench until the second quarter. The coaches rationalize this move by stating that the player's minutes now are less important than his minutes later, and we do not want to risk losing those later minutes by having him foul out now. This rule is called the Q+1 rule. Moskowitz says coaches should leave the player in the game more often than not, despite the foul trouble.[26] Maymin and Shen (2011) use a win-probability model to argue that the coaches are right, and the Q+1 rule is right. [27]

The first paper sets the probability of the home team winning as $N(F_t/\sigma_t)$, where N is the cdf of the standard normal distribution, based on a model by H. Stern. The game goes from 0 to the end, where $0 \leq t \leq 1$, t represents the fraction of time elapsed. The variance is proportional to $1-t$ so that $\sigma_t^2 = (1-t)\sigma^2$ for some volatility constant σ, and F_t is what they define as the forward lead at time t:

$$F_t = \beta_l l_t + (1-t)\mu \tag{12}$$

In this paper, Maymin and Shen are trying to assess the impact of a coach removing a player when he gets a foul past a general $Q+1$ decision rule (where Q is the quarter; i.e. a player in the 2nd quarter is removed after receiving his third foul). They define new variables:

STA: net number of starters in the game

[24] Hollinger, John. Pro Basketball Forecast: 2005-2006. Potomac Books Inc., 2005.

[25] *ibid*

[26] Moskowitz, Tobias, and L. Jon Wertheim. Scorecasting: The Hidden Influences Behind How Sports Are Played and Games Are Won. Reprint. Three Rivers Press, 2012.

[27] Maymin, Allan, Philip Maymin, and Eugene Shen. How Much Trouble Is Early Foul Trouble? Strategically Idling Resources in the NBA. SSRN Scholarly Paper. Rochester, NY: Social Science Research Network, January 8, 2011. http://papers.ssrn.com/abstract=1736633.

FTR: net number of starters in the game with foul trouble, where foul trouble is defined

They add to the Stern model by adding some other variables, possession, team dummies, a constant, starter and foul variables:

$$F_t = \alpha + \beta_l l_t + \beta_p P_t + (1-t)\left(\mu + \sum_{i=1}^{29} \beta_i D_{it} + \beta_{STA} STA_t + \beta_{FTR} FTR_t\right) \tag{13}$$

where α is a constant, P_t represents possession at time t, β_p is its coefficient, β_{STA} and β_{FTR} are coefficients associated with STA and FTR, and D_{it} is a dummy variable for each NBA team (besides one which is normalized to zero), and β_i is their coefficient.

They set STA and FTR for the beginning of the game when each team has 5 starters in no foul trouble:

$$STA(home) = STA(away) = 5 \implies STA = 0$$

$$FTR(home) = FTR(away) = 0 \implies FTR = 0$$

They define what they are looking for and expect:

> If the home team has a starter in foul trouble, then the coach faces the following choice: either keep the starter in the game (hence, increment FTR), or yank the starter from the game (hence, decrement STA but leave FTR unchanged).
>
> We expect β_{STA} to be positive, since teams play better if their starters are in the game. The optimality of yanking will depend on the size of β_{FTR}. If $\beta_{STA} + \beta_{FTR} < 0$, then it will be optimal to yank, since the starter in foul trouble plays worse than a bench player. If $\beta_{STA} + \beta_{FTR} > 0$, then it may be suboptimal to yank locally, but it may still be optimal to yank globally since yanking preserves option value by reducing the probability that the player remains in foul trouble.

They find, after analyzing their model with data from 6 NBA seasons, that most of the time, a starter in foul trouble should be taken from the game.

David Annis uses a decision tree to analyze whether to foul or not when a team has a three point lead.[28] Annis examines two possible strategies for a defensive team who is leading by three points with the shot clock turned off. He calls them the non-Few or Few strategy. [29] In the non-Few strategy the defensive team fouls the offensive team and sends them to the line for

[28] Annis, David H. Optimal End-Game Strategy in Basketball. Journal of Quantitative Analysis in Sports 2, no. 2 (2006). http://ideas.repec.org/a/bpj/jqsprt/v2y2006i2n1.html.

[29] named after Gonzaga coach Mark Few.

the two free throws. There are four possible outcomes: if they make the first free throw then they will purposely miss the second and either tip it in [30] to force overtime, or if anything else occurs the defensive team will win. If he misses the first free throw throw, either they will get an offensive rebound and shoot a three for the tie, or anything if else occurs, the defensive team will win.

Figure 1:

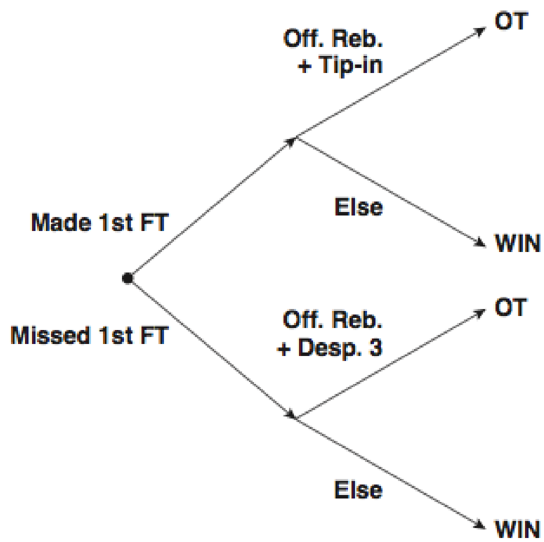
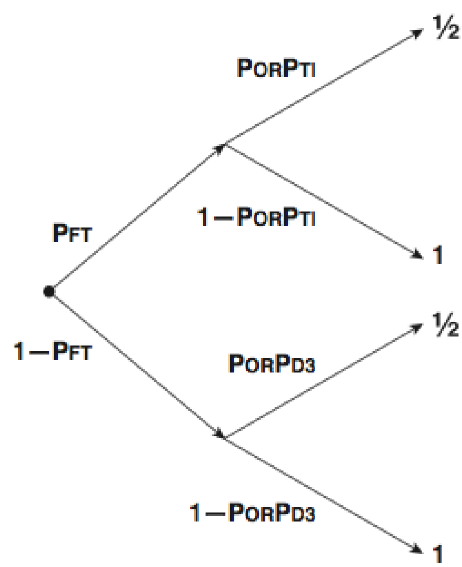


Figure 2:



[30]he assumes they will not shoot a three here, but rather go for the tie, I will also look at chances they make a three there and the fouling team loses

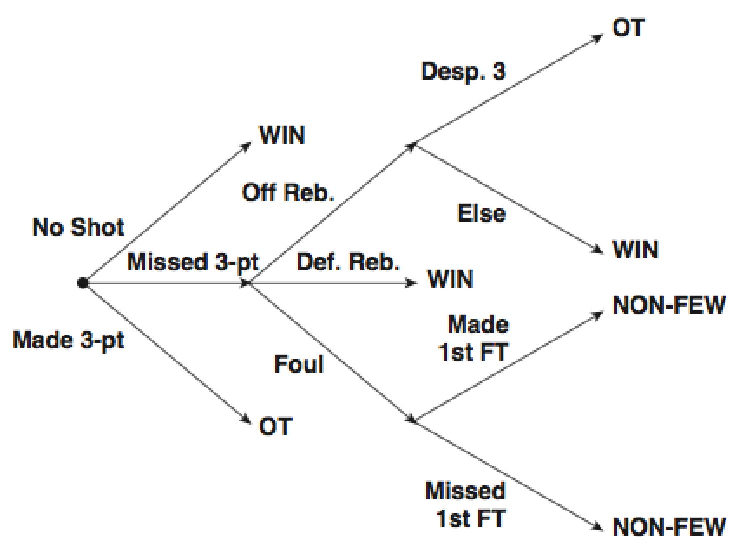
He shows the defensive team's probability of winning the game under the non-Few strategy to be:

$$p_{nF} = p_{FT}\left[\frac{1}{2}p_{OR}p_{TI} + (1 - p_{OR}p_{TI})\right] + (1 - p_{FT})\left[\frac{1}{2}p_{OR}p_{D3} + (1 - p_{OR}p_{D3})\right] \tag{14}$$

> where p_{FT} is the probability of a made free throw, p_{OR} is the probability of an offensive rebound after a missed free throw,p_{TI} is the probability of converting a subsequent tip-in and p_{D3} is the probability of converting a desperation three- point basket.

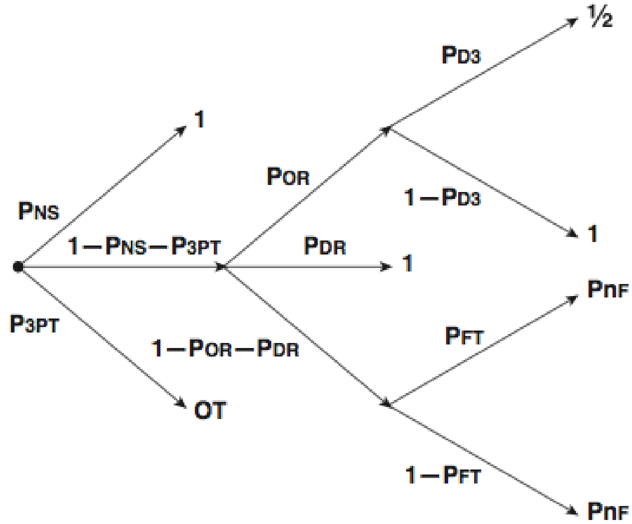
Using a similar analysis, he calculates the probability of winning the game given the Few strategy, with seven possible outcomes. Here the defensive team does not foul. Three things could initially occur: the offense could make the shot and tie the game, the offense could not get off a shot, and the defensive team would then win, or miss the shot. If the offense misses the shot, the defense could get a rebound and win, or the offense could get an offensive rebound and either tie the game on a desperation three or if anything else occurs the defensive team would win, or the defense could then foul and then we would be the non-Few strategy tree.

Figure 3:



$$p_F = p_{NS} + \frac{1}{2}p_{3pt} + (1 - p_{NS} - p_{3pt}) * \left[p_{OR}\left(\frac{1}{2}p_{D3} + (1 - p_{D3})\right) + p_{DR} + (1 - p_{OR}p_{DR})p_{nF}\right] \tag{15}$$

Figure 4:



where p_{NS} is the probability that the offense attempts no shot, p_{3pt} is the probability of the offensive team converting a three-point attempt and p_{DR} is the probability of a defensive rebound after a missed shot.

Given certain assumptions Annis is able to conclude that $p_{nF} > p_F$: teams should always foul. His method of assigning a probability to each play could easily be expanded by adding more possible outcomes by taking away a few assumptions. Annis also does not use data to calculate the probabilities, he uses reasonable assumptions for the probability of every play

3 Theory and Models

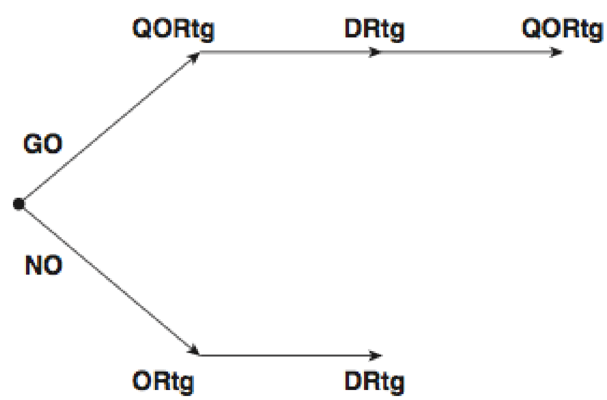
3.1 Two-for-one

Using the same decision tree framework model, I attempt to estimate the probabilities for the two-for-one situation. The decision is to go two-for-one or not. The tradeoff can be made using Oliver's offensive rating and defensive rating formula. Recall equation 3:

$$ORtg_t = \frac{PTS_t}{POSS_t} * 100 \qquad DRtg_t = \frac{PTS_O}{POSS_O} * 100 \tag{16}$$

The tradeoff is evaluated using these metrics. I need to create a new $ORtg$ for quick shots, which I will call quick offensive rating: $QORtg$. The decision about going two-for-one needs to be evaluated at the point differential level. Will going two-for-one score me more points and my opponent less points than not? That is the essential question modeled below:

Figure 5:



Here we see your expected point differential for not going two-for-one is equal to:

$$DIF_{NO} = ORtg - DRtg \tag{17}$$

You each get one possession, where you score on average your offensive rating. Your opponent gets one possession where they score on average your defensive rating. When you go two-for-one your expected point differential is:

$$DIF_{GO} = 2QORtg - DRtg \tag{18}$$

You get two possessions, where you score on average your quick offensive rating. But we multiply by two because you get two possessions, and we subtract out the points you give up: the defensive rating. I use the quick offensive rating because you are trading the one well developed play for the two quick offensive plays.

You should go for two-for-one when: $DIF_{GO} > DIF_{NO}$. That is true when:

$$2QORtg - DRtg > ORtg - DRtg \tag{19}$$

$$QORtg > \frac{ORtg}{2} \tag{20}$$

So according to this model you should go for two-for-one when your quick offensive rating is greater than half of your normal offensive rating. This model assumes independence of defensive possession from offensive possession; whether you go for it or not is independent of how the defensive possession will turn out. The last assumption this model makes is that offensive and defensive possessions are finite - you cannot go for a shot, get an offensive rebound and then keep getting offensive rebounds until the quarter ends. In other words, you have an offensive possession, and you either score or do not; and then your opponent has an offensive possession and they score or do not. When you run a quick play and get two offensive possessions, and one defensive possession - that is going two-for-one. When you run a well developed play and have one offensive possession and one defensive possession you have not gone two-for-one.

We can further calculate the minimum percent chance every shot that goes in needs to have. Oliver's offensive rating is per 100 possessions, so if we divide it by 100, we get points per possession, which we need to be less than the expected points on the quick shot.

$$\frac{ORtg}{2 * 100} = \text{points} * \text{probability of made shot} \tag{21}$$

For a 2-point shot or 3 point:

$$\frac{ORtg}{2 * 100} = 2 * p_2 \qquad \frac{ORtg}{2 * 100} = 3 * p_3$$

$$p_2 = \frac{ORtg}{400} \qquad p_3 = \frac{ORtg}{600} \tag{22}$$

[31]

3.2 Intentionally Fouling Up Three

I use the Annis' model as my model of fouling up three. Comparing equations: 14 and 15, to see which strategy has a higher probability of leading to a win. They are described in Section 2, but below are the equations that describe the probability of winning.

$$p_F = p_{NS} + \frac{1}{2}p_{3pt} + (1 - p_{NS} - p_{3pt}) * \left[p_{OR}\left(\frac{1}{2}p_{D3} + (1 - p_{D3})\right) + p_{DR} + (1 - p_{OR}p_{DR})p_{nF}\right]$$

$$p_{nF} = p_{FT}\left[\frac{1}{2}p_{OR}p_{TI} + (1 - p_{OR}p_{TI})\right] + (1 - p_{FT})\left[\frac{1}{2}p_{OR}p_{D3} + (1 - p_{OR}p_{D3})\right]$$

You will foul intentionally when: $p_{nF} > p_F$

[31] This assumes that in each of the quick offensive shots you would take two three-pointers, or two-pointers. This formula doesn't apply if you take two different types of shots

4 Data

I used data from STATS Inc. for four NBA Seasons: 2002-03 through 2005-06. The data was organized by event. Every event during a basketball game – shot, free throw attempt, foul, jump ball, violation, etc... – was recorded and time-stamped. A possession could be either a series of events or just one event, depending on the possession. I used Stata to organize and analyze the data.

4.1 Two-for-one

The two-for-one model ended up relying on the differences in *ORtg*. Below is the average *ORtg* for the top 5 and bottom 5 NBA teams in 2005-6 year:[32]

Top 5		Bottom 5	
1 Phoenix	96.4	30 Portland	84.3
2 Detroit	95.6	29 New York	85.2
3 Dallas	94.6	28 Houston	85.3
4 Toronto	93.8	27 Utah	93.8
5 Seattle	93.0	26 Minnesota	86.1

[33]

The criterion for two-for-one being the optimal strategy was that the quick offensive rating be greater than half of the normal offensive rating. The *QORtg* for our best offensive team, Phoenix, would need to be 48.2 i.e. Phoenix would have to score more .482 points per possession. That means Phoenix needs to be shooting two-pointers with a 24% chance of going in, and three-pointers with a 16% chance of going in. As a team's offensive rating gets worse, the chances of the shots going in can get worse. Portland only needs to have a $QORtg = 42.5$ and shoot two two-pointers at a 21% rate, or two three-pointers at a 14% rate. These are not high thresholds in basketball to achieve.

In the event-by-event data, I look at all situations when a team goes 2 for 1. I isolate all strings of plays under 48 seconds when teams went two-for-one. I trimmed the data to only look at situations in the first three quarters with under 48 seconds.[34] I counted all possessions by

[32]The last year of my data set
[33]http://espn.go.com/nba/hollinger/teamstats
[34]I didn't include the fourth quarter because there is too much noise from all the fouling

both teams in the data, starting with the first full shot clock possession. I also did not count possessions that started with under a second left in the quarter. I justify those as not counting as possessions by with the following chart describing shot distance.

Time	Attempts	Mean	Std. Dev
Time > 1	28648	10.58692	10.69709
Time < 1	9895	22.02527	18.57363

The average distance of a shot with under a second left is almost twice that of all other shots in the quarter. Those last second shots are much more likely to be desperation attempts that only make it because there is a little bit of time on the clock. When I later calculate the net difference in points for going for two-for-one or not, I include any points scored in the last second; but I do not count the last second as a possession. The net number of possessions is below:

# of Poss	Obs	%
1	2250	14.96
2	6286	41.79
3	4718	31.37
4	1360	9.04
5	255	1.7
6	36	0.24
7	6	0.04
< 1 sec	131	0.87
Total	15042	100

For the last 48 seconds a plurality of the data is two possessions- one per team. Around 44% of the time teams have at least a two-for-one. While intent is nearly impossible to retrieve from the data, I define any attempt at two-for-one to be when there are more than three possessions i.e. the offense got at least two possessions.

The average two-point are$FG\% = 43\%$ and three-point are $FG\% = 23\%$. These are above the barrier set by the offensive ratings calculated above.

Going two-for-one seems to be worth it by the $ORtg$ measure. But I also looked at net point differential for when teams go two-for-one and when they do not. Here I summed across all points for the team that had the opportunity to go two-for-one, and subtracted the points scored by their opponents.

Net Points	Mean	Std. Err	Std. Dev.	Obs	t-test for difference
Not Going 2:1	-0.1571178	0.0173352	1.60151	8535	$t = -16.9434$
Going 2:1	0.3563413	0.0248564	2.004761	6505	df = 12174.5
Difference	0.5134591		Total:	15040	

I find significant difference, almost half a point, from going for it as opposed to not. Running a t-test, we see this difference is very significant. Surprisingly, not going two-for-one has a negative point differential. This holds true for all seasons, and for all the robustness tests I ran. Since not going two-for-one includes all the situations where the team just had one possession on offense and possibly a second, I intuitively would expect the not going two-for-one point differential to be zero or slightly positive.

In the appendix are the tests of robustness, where I change the times of the last possession, and when I start counting the attempt for two-for-one. I also run each test for individual seasons, not just the whole data set. Below is the summary for individual seasons:

Year	Difference	t-test
2002-3	0.4618125	-7.5943
2003-4	0.5827336	-10.0157
2004-5	0.4915201	-8.0038
2005-6	0.5224837	-8.4499

4.2 Intentionally Fouling Up Three

Estimating the exact probabilities that make up the Annis model is difficult, because there are so few data points. There were only 607 games over the four seasons of data that met the criteria of the defense being up three with the shot clock turned off.

Annis estimated the model using what he determined as reasonable probabilities for each type of event occurring.

Because of the dearth of data and the difficulty of determining intent of plays, I am not confident with the probabilities I can get from my data to estimate the model. I decided to simply look at outcomes in the 607 games that met the criteria. I only looked at situations where the defensive team was up three and had to decide to foul under 10 seconds. I limited the data to 10 seconds because I wanted to have situations where the offense knows that any shot is likely to be their last shot of the game, so they will be trying to tie the game with a three. At more than 10 seconds there is a possibility that the offense will attempt a quick two-point shot, and then foul to stop the clock. At under 10 seconds there is only time for one last shot, which must be a three-pointer.

I defined from the data when a team was fouling or not by assigning any foul that led to two-free throws as an intentional foul. While in the data it is possible to determine the difference between a foul committed while a shooter is shooting, and the basket goes in (and-one), there is no way to determine whether a player was intentionally fouled, or fouled shooting and the shot missed. I had to assume that all fouls that led to two free-throws were intentional fouls.

Here is a summary of the results:

Foul and win	7
Foul and lose	0
Foul and tie	1
No foul and win	567
No foul and lose	6
No foul and tie	26
Total	607

Of the 607 games, teams only intentionally fouled 8 times. 7 times the fouling team won. 1 time it went into overtime.[35] While this is not overwhelming evidence, it does support intentionally fouling as a strategy. The counterfactual, not fouling, occurred 599 times. The team that decided not foul still won 95% of the time. But 6 times the team lost, and 26 times the game went to overtime.[36] The counterfactual shows the obvious, not fouling puts your time in a position to have the game go to OT or lose. While it is not frequent, it can happen. Perhaps most surprisingly, teams intentionally fouled only 8 times in the 607 games. Despite a team never losing when they intentionally fouled, the 8 games are not enough to show that intentionally fouling is definitively the right strategy.

[35] The team that was ahead by and decided not foul went 1-0 in OT.

[36] The team that was ahead by and decided not foul went 16-10 in OT.

5 Summary and Conclusions

In a basketball game, more possessions mean more opportunities to score. I find in the data a significant difference in net scoring differential when you go two-for-one than when you do not. This is backed up by looking at the minimum offensive rating requirements to make going two-for-one worth it.

Given this evidence, why would a team not want to go two-for-one? Former NBA head coach Stan Van Gundy said at the Sloan Conference that he believed from the data that going two-for-one was worth it, but he still did not always go for it. He said that he would only do it if he had a smart point guard who could recognize situations where it was appropriate. Van Gundy was afraid of the spillover affects of getting his players into the habit of taking a rushed shot at certain points in the game. [37]

This is a legitimate fear. Can players successfully detect the two-for-one situation; and even if they can, will there be any effects at other points in the game. Will the lack of a developed play in those situations somehow affect other plays where the team will now forget or get out of the habit of running the normal plays? These are legitimate concerns that probably only a coach can determine if his players are able to handle the switching of strategies. Van Gundy said he would absolutely go two-for-one if he had a smart point guard like Chris Paul.

There are many ways to push this research further. Using Van Gundy's idea of differences in intelligence level of point guards, we should classify all point guards in the league and then test correlations between smart point guards and successfully running two-for-one.

Also, further research should make a more dynamic model, where teams can respond to an attempt to go two-for-one. My model assumes that when you go two-for-one the other team's behavior is static. But we know that teams can adjust behavior, and newer models should try to account for this. Teams might change their defense in response to knowing that the opponent will go for a quick shot. Teams may also try to go "three-for-two" if they know that the opponent will try and go two-for-one. Game theoretic models that take into account multiple rounds, and responses, will be helpful in analyzing these situations.

Another issue I did not consider was pace. Pace can affect how different teams would

[37] Basketball Analytics 2013 — MIT Sloan Sports Analytics Conference. March 2, 2013. http://www.sloansportsconference.com/?p=9994.

approach going two-for-one. A team that is accustomed to a fast pace would not have a big drop off between their normal offensive rating and their quick offensive rating. Accounting for pace would give a clearer picture of the heterogeneity of outcomes of going two-for-one across different teams. A possible result is that teams used to going at a fast pace always should go two-for-one, because their quick offensive rating is probably closer to their normal offensive rating, and other slower teams shouldn't go two-for-one.

Analytics should provide the clear information coaches need to design and implement strategy. In this data set the analytics are clear: going two-for-one is worth it. How the coaches make decisions from this evidence is their choice, and as Van Gundy has set forth, there are good reasons to not go for it even if the analytics supports it. But a good reason to not go for it cannot be because you think you will, in these specific situations, score less than the other team. The data are convincing that if you go two-for-one you will outscore your opponent in those situations.

Giving another team a free opportunity to score might not seem like the optimal strategy, but it may be. The data, while certainly not overwhelmingly convincing, show that when you intentionally foul you do not lose, but when you do not intentionally foul you might go to overtime, and you might even lose.

In order to lose when fouling a number of rare things must occur. The offensive team must make their first free-throw, and then must purposely miss the second free-throw, get the offensive rebound, and then not go for the tie, but rather hit a desperation three to win. Even if the offensive team gets the offensive rebound, they still might just tie the game on a two-pointer and send it to overtime.

In an interview with the *New York Times* about fouling when intentionally up three, Boston Celtics Coach Doc Rivers discusses some of the troubles that players have with executing a plan that coaches set up. "We literally forgot to foul," Rivers said. "We came out of the timeout and were going to foul. We messed it up. They score and win the game in overtime." [38]

Other coaches, like then-Atlanta Hawks coach Mike Woodson, say they never foul: "I never take the foul, I just always put it on our team to defend." According to the *New York Times*

[38] Abrams, Jonathan, and Howard Beck. Fouling to Protect a Lead: Debated but Rarely Done. The New York Times, April 21, 2010, sec. Sports / N.B.A. http://www.nytimes.com/2010/04/22/sports/basketball/22fouls.html.

article:

> Coaches weigh several factors before deciding to foul intentionally: the time left, the timeouts for both teams, the foul situation, the likely ball handlers, defensive rebounding and offensive rebounding, where the ball is inbounded, the experience of the players involved, both teams free-throw shooting abilities, whether they can successfully execute the foul and how much the team has practiced that situation.

Intentionally fouling might not be the right strategy across all NBA games, but rather dependent on which teams and players are on the floor. While coaches may not want to foul a team with excellent offensive rebounders, they probably want to foul a team with excellent three-point shooters. The parameters of the Annis model change on the composition of players on the court.

This paper attempts to add to the information coaches use to make seemingly simple decisions. The more information coaches have about strategy, the closer they can get to making the best decision. Sports analytics provides context and more information, but as variables in the game change, a coach must make decisions. Analytics should not try to give black-and-white answers, but rather help clear up the very muddy picture basketball coaches face when making decisions.

6 Appendix A

6.1 Other Strategic Decisions

This unique event-by-event data set could answer a series of other questions.

1. Is Hack-a-Shaq effective?
   On defense, teams will foul players who are poor free throw shooters. This is another tradeoff. Teams are giving up two low percentage free-throws, so that the offense doesn't get a higher percentage shot.

2. Down 2, with no shot-clock, should you go for the tie or win?
   On offense, knowing that this will be the last shot of the game, should teams go for a two-point field goal, and hope to win in overtime. Or should teams go for the win now.

3. Is the quarter plus one foul rule strategically the best decision for a coach to make?
   This question has been explored by Moskowitz, and others. It is further described in section II of this paper.

4. What is the optimal possession time for an offense?
   How long should the offense take to run a play. In the NBA we've seen teams that take the full shot clock but also some teams have tried to shoot really quickly i.e. Mike D'Antoni's "seven seconds or less" offense.

5. When is the right time for coaches to call timeouts? When should coaches call timeouts? Should they use timeouts to stop runs by the opposing team? Should they use it to setup play? How successful are set plays out of timeouts?

6. How should coaches reconfigure lineups for particular match-ups? Some teams "go-big" playing multiple tall players, other teams go "small". How should teams adjust their own lineups to counter, if at all? What lineups, big or small, fast or strong, etc.. are the best?

7 Appendix B

7.1 Stata code

7.1.1 Two-for-one

```
insheet using "plays0203.txt", comma clear
save basketball.dta, replace

insheet using "plays0304.txt", comma clear
append using basketball.dta
save basketball.dta, replace
```

```
insheet using "plays0405.txt", comma clear
append using basketball.dta
save basketball.dta, replace

insheet using "plays0506.txt", comma clear
append using basketball.dta
save basketball.dta, replace

rename v1 gamecode
rename v2 index
rename v3 offense
rename v4 defense
rename v5 off_player1
rename v6 off_player2
rename v7 def_player
rename v8 event
rename v9 detail
rename v10 points
rename v11 period
rename v12 time
rename v13 oncourt
rename v14 off_score_old
rename v15 def_score
rename v16 off_score
rename v17 distance

*getting rid of subs, time outs, ejections, violations
drop if (event==10|event==11|event==13|event==9|event==14|event==15)

*get rid of fouls to clear up possessions
drop if (event==8)

*get rid of team offensive rebounds from fouls (orb between free throws)
gen torb = (event==5 & off_player1==0 & event[_n-1]==2 & (event[_n+1]==1 | event[_n+1]==2))
drop if torb == 1
drop torb

*Create a dummy for all possessions
gen poss = offense != offense[_n-1] if time >"0:00:03"
gen posscount = 1
replace posscount = posscount[_n-1] + poss if gamecode == gamecode[_n-1] & period ==period[_n-1]
egen possfin = max(posscount), by(gamecode period)
gen poss21 =1 + possfin - posscount

*Select 2 for 1 situations.
keep if period < 4 & time <= "0:00:48"

*get rid of jump ball chains which complicate data
gen flag1 = event== 12
egen periodid = group(gamecode period)
egen flag = max(flag1), by(periodid)
drop if flag == 1
drop flag1 flag

gen beginperiod = period !=period[_n-1]

*Make sure we start with first full offensive possession under 48 seconds
```

```
egen maxpos = max(poss21) if beginperiod==1,by(periodid)
egen maxpos1 = max(maxpos), by(periodid)
gen tag = poss21 == maxpos1
drop if tag==1
drop maxpos1 maxpos tag
drop beginperiod
gen beginperiod = period !=period[_n-1]
gen endperiod = period !=period[_n+1]

*find out which team starts on offense
gen testteam = offense if beginperiod==1
gen dtestteam = defense if beginperiod==1
egen offteam = max(testteam), by(periodid)
egen defteam = max(dtestteam), by(periodid)
drop testteam dtestteam

*generate starting and ending differences
gen dif = off_score - def_score
replace dif = -dif  if defense == offteam

gen dife = dif*endperiod
gen difb = dif*beginperiod
replace dife = . if dife ==0 & endperiod != 1
replace difb = . if difb ==0 & beginperiod != 1
egen dife_fil = max(dife), by(periodid)
egen difb_fil = max(difb), by(periodid)

*generate net change in differences
gen netdif = dife_fil - difb_fil

*create flag for 2 for 1 attempts
gen flag21 = poss21 >= 3

*RESULTS:
tab flag21 if beginperiod==1, sum(netdif)
tab poss21 if beginperiod==1, m
ttest netdif if beginperiod == 1, by(flag21) unequal
```

7.1.2 Intentionally Fouling Up Three

```
rename v1 gamecode
rename v2 index
rename v3 offense
rename v4 defense
rename v5 off_player1
rename v6 off_player2
rename v7 def_player
rename v8 event
rename v9 detail
rename v10 points
rename v11 period
rename v12 time
rename v13 oncourt
rename v14 off_score_old
rename v15 def_score
rename v16 off_score
```

```
rename v17 distance

*getting rid of subs, time outs, ejections, violations
drop if (event==10|event==11|event==13|event==9|event==14|event==15)

*get rid of team offensive rebounds from fouls (orb between free throws)
gen torb = (event==5 & off_player1==0 & event[_n-1]==2 & (event[_n+1]==1 | event[_n+1]==2))
drop if torb == 1

*Generate dummy for field goal attempt and field goal make, fouls, and free throws
gen fga = (event == 3 | event == 4)
gen fgm = (event == 3)
gen fouls = (event ==8)
gen fta = (event == 1 | event ==2)

*generate differential (positive for team on offense)
gen dif = def_score-off_score

*create dummy for last play of game
gen endgame = gamecode !=gamecode[_n+1]

*creating an end of game dummy for when the offense wins
gen w = off_score-def_score > 0
gen offwin = endgame * w

*creating an end of game dummy for when the defnese wins
gen d = def_score-off_score > 0
gen defwin = endgame * d

*test records to see if data matches with season records
gen winningteam = offense*(offwin == 1) + defense*(defwin == 1)
gen losingteam = offense*(defwin ==1) + defense*(offwin==1)
egen winteam = max(winningteam), by(gamecode)
egen loseteam = max(losingteam), by(gamecode)

*generate ties at end of periods or OT
gen endperiod = period !=period[_n+1]
gen ties = def_score == off_score & endperiod==1

*flagging all chains of end of game situations where the offensive team has a lead of 3
gen dummy = period >= 4 & time <= "0:00:10" & dif ==3
egen periodid = group(gamecode period)
egen flag = max(dummy), by(periodid)
keep if flag == 1 & time < "0:00:10"

*identifier key:
*1= field goal attempts
*2= 2 pt field goal made
*3= 3 pt fgm
*4= turnovers
*5= bonus fouls
*6=  and 1 on 2 pointer
*7=  and 1 on 3 pointer
*15= fouled shooting 2
*9= fouled shooting 3

#d ;
```

```
gen identifier =
(dif == 3)*(fga == 1)*(abs(dif[_n-1])==3) +
2*(dif==1)*(fgm == 1 & points == 2) +
3*(dif == 0)*(fgm ==1)*( points == 3) +
4*(dif == 3)*(event == 7) +
5*(dif==-3)*(event == 8)+
4*(dif==1)*(fgm == 1 & points == 2)*(event[_n+1]==8)*(detail[_n+2]==10)+
4*(dif == 0)*(fgm ==1)*( points == 3)*(event[_n+1]==8)*(detail[_n+2]==10) +
10*(dif==-3)*(event == 8)*(detail[_n+1]==11) +
4*(dif==-3)*(event == 8)*(detail[_n+1]==13);
#d cr

*find out which team was winning at start, and whether they fouled or not
gen winning_at_start = defense*(dif > 0) + offense*(dif < 0) if gamecode != gamecode[_n-1]
egen winning_at_start_fillin = max(winning_at_start), by(gamecode)
gen fiveindicator = identifier == 15 & winning_at_start == offense
egen fiveever = sum(fiveindicator), by(gamecode)

*five and win team is offense
gen fiveandwin = endperiod == 1 & fiveever == 1 & ties!=1 &winning_at_start_fillin == winteam
gen fiveandlose = endperiod == 1 & fiveever == 1 & ties!=1 &winning_at_start_fillin == loseteam
gen fiveandtie = endperiod == 1 & fiveever == 1 & ties ==1 & winning_at_start_fillin !=0
gen fivetiewin = endperiod == 1 & fiveever == 1 &  ties ==1 & winning_at_start_fillin == winteam
gen fivetielose= endperiod == 1 & fiveever == 1 &  ties ==1 & winning_at_start_fillin == loseteam

gen nofiveandwin = endperiod == 1 & fiveever == 0 & ties!=1 & winning_at_start_fillin == winteam
gen nofiveandlose = endperiod == 1 & fiveever == 0 & ties!=1 & winning_at_start_fillin == loseteam
gen nofiveandtie = endperiod == 1 & fiveever ==0 & ties==1 & winning_at_start_fillin !=0
gen nofivetiewin = endperiod == 1 & fiveever == 0 &  ties ==1 & winning_at_start_fillin == winteam
gen nofivetielose= endperiod == 1 & fiveever == 0 &  ties ==1 & winning_at_start_fillin == loseteam

*results
sum *five* if endperiod==1

*Estimate probabilities for Annis formula:
tab events
```

7.2 Robustness and Detailed Data

Below shows that changing any of the decisions of when to cut off data, does not significantly effect my results for both models. Also are the breakdowns for every season, not just the year-by-year data

7.2.1 Two-for-one

Changing possession cutoff to 0

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.1678667	0.0185858	1.564188	7083	t = -15.1883
Going 2:1	0.2708648	0.0221128	1.972383	7956	df = 14843.5
Difference	0.4387315		Total	15039	

# of Poss	Obs
1	1528
2	5556
3	5378
4	1990
5	440
6	61
7	9
8	1
< 1 sec	78
Total	15041

Not making a possession cutoff

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.146734	0.0259888	1.463009	3169	t = -8.5458
Going 2:1	0.1200236	0.0172906	1.882688	11856	df = 6265.21
Difference	0.2667576		Total	15025	

# of Poss	Obs
1	400
2	2769
3	5874
4	4324
5	1339
6	274
7	41
8	7
Total	15028

Changing starting time to 49

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.1769857	0.0179068	1.612407	8108	t = -17.3753
Going 2:1	0.341921	0.0242787	2.020684	6927	df = 13179
Difference	0.5241778		Total	15035	

# of Poss	Obs
1	2016
2	6094
3	4936
4	1538
5	296
6	44
7	6
< 1 sec	108
Total	15038

Changing starting time to 47

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.154162	0.0167343	1.588436	9010	t = -17.2063
Going 2:1	0.371457	0.0255567	1.985047	6033	df = 10963.9
Difference	0.525619		Total	15043	

# of Poss	Obs
1	2517
2	6494
3	4459
4	1170
5	220
6	28
7	5
$<$ 1 sec	152
Total	15045

Year breakdown, normal constraints:

2002-3:

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.164272	0.0344998	1.576458	2088	$t = -7.5943$
Going 2:1	0.2975405	0.0500769	2.044584	1667	df = 3070.6
Difference	0.4618125		Total	3755	

# of Poss	Obs
1	517
2	1571
3	1172
4	380
5	76
6	7
7	2
$<$ 1 sec	30
Total	3755

2003-4:

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.1549558	0.0335938	1.557316	2149	$t = -10.0157$
Going 2:1	0.4277778	0.0475036	1.911984	1620	df = 3065.41
Difference	0.5827336		Total	3769	

# of Poss	Obs
1	591
2	1558
3	1183
4	332
5	65
6	3
7	3
<1 sec	34
Total	3769

2004-5

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.1681985	0.0351168	1.638115	2176	$t = -8.0038$
Going 2:1	0.3233216	0.0503794	2.075973	1698	df = 3163.91
Difference	0.4915201		Total	3874	

# of Poss	Obs
1	571
2	1606
3	1232
4	356
5	60
6	13
7	1
<1 sec	37
Total	3876

2005-6

Net Points	Mean	Std. Err	Std. Dev	Obs	t-test for difference
Not Going 2:1	-0.1409048	0.0354461	1.63283	2122	$t = -8.4499$
Going 2:1	0.3815789	0.0506657	1.975272	1520	df = 2876.4
Difference	0.5224837		Total	3642	

# of Poss	Obs
1	571
2	1551
3	1131
4	292
5	54
6	13
7	0
<1 sec	30
Total	3642

7.2.2 Intentionally Fouling Up Three

Change time to 24.

Foul and win	7
Foul and lose	0
Foul and tie	0
No foul and win	780
No foul and lose	26
No could and tie	59
Total	872

OT Record: 30-29

Change time to 15.

Foul and win	6
Foul and lose	0
Foul and tie	1
No foul and win	669
No foul and lose	13
No could and tie	36
Total	725

OT Record: Foul: 1-0 No Foul: 20-16

Change time to 5.

Foul and win	7
Foul and lose	0
Foul and tie	0
No foul and win	446
No foul and lose	1
No could and tie	10
Total	464

OT Record: 4-6

8 References

About the Author. http://www.basketballonpaper.com/author.html.
Abrams, Jonathan, and Howard Beck. Fouling to Protect a Lead: Debated but Rarely Done. The New York Times, April 21, 2010, sec. Sports / N.B.A.
http://www.nytimes.com/2010/04/22/sports/basketball/22fouls.html.
Annis, David H. Optimal End-Game Strategy in Basketball. Journal of Quantitative Analysis in Sports 2, no. 2 (2006). http://ideas.repec.org/a/bpj/jqsprt/v2y2006i2n1.html.
Baseball Prospectus — Lies, Damned Lies: Introducing ORVY. Baseball Prospectus. Accessed May 3, 2013. http://www.baseballprospectus.com/article.php?articleid=4003.
Baseball Prospectus — Schrodingers Bat: Wins and the Quantum. Baseball Prospectus. Accessed May 3, 2013. http://www.baseballprospectus.com/article.php?articleid=4945.
Basketball Analytics 2013 — MIT Sloan Sports Analytics Conference. Accessed May 3, 2013.
http://www.sloansportsconference.com/?p=9994.
Coloma, Germn. Penalty Kicks in Soccer An Alternative Methodology for Testing Mixed-Strategy Equilibria. Journal of Sports Economics 8, no. 5 (October 1, 2007): 530545. doi:10.1177/1527002506289648.
Chiappori, P.-A., S. Levitt, and T. Groseclose. 2002. "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer ." The American Economic Review, 92(4): 1138-1151.
Dodes, Rachel. Older, Mellower, but Still Woody. Wall Street Journal, June 15, 2012.
http://online.wsj.com/article/SB10001424052702303822204577466490672066730.html.
Hollinger, John. Pro Basketball Forecast: 2005-2006. Potomac Books Inc., 2005.
−− Pro Basketball Prospectus: 2003 EDITION. F. Potomac Books, 2003.
John Hollinger — MIT Sloan Sports Analytics Conference. Accessed May 3, 2013.
http://www.sloansportsconference.com/?p=414.
Kovash, Kenneth, and Steven D. Levitt. Professionals Do Not Play Minimax: Evidence from Major League Baseball and the National Football League. NBER Working Paper. National Bureau of Economic Research, Inc, 2009. http://ideas.repec.org/p/nbr/nberwo/15347.html.
Lewis, Michael. Moneyball. W. W. Norton & Company, 2011.
Maymin, Allan, Philip Maymin, and Eugene Shen. How Much Trouble Is Early Foul Trouble? Strategically Idling Resources in the NBA. SSRN Scholarly Paper. Rochester, NY: Social Science Research Network, January 8, 2011. http://papers.ssrn.com/abstract=1736633.
Moskowitz, Tobias, and L. Jon Wertheim. Scorecasting: The Hidden Influences Behind How Sports Are Played and Games Are Won. Reprint. Three Rivers Press, 2012.
Myerson, Roger B. Game Theory: Analysis of Conflict. Harvard University Press, 1997.
Oliver, Dean. Basketball on Paper: Rules and Tools for Performance Analysis. Potomac Books Inc., 2004.
Pelton: What I Learned at Sloan. ESPN.com. Accessed May 3, 2013.
http://insider.espn.go.com/nba/story/_/page/PerDiem-130304/nba-key-points-mit-sloan-sports-analytics-conference.
Romer, David. Do Firms Maximize? Evidence from Professional Football. Journal of Political Economy 114, no. 2 (2006): 340365.
Silver, Nate. Baseball Prospectus — Lies, Damned Lies: The PECOTA Top 100. Baseball Prospectus. Accessed May 3, 2013. http://www.baseballprospectus.com/article.php?articleid=6153.
The Kobe Assist. Grantland.com. Accessed May 3, 2013.
http://www.grantland.com/story/_/id/8719297/how-kobe-bryant-missed-shots-translate-new-nba-statistic-kobe-assist.
Up Three, Without the Ball: To Foul or Not? New Insight. ESPN.com. Accessed May 2, 2013. http://espn.go.com/blog/truehoop/post/_/id/6992/up-three-without-the-ball-to-foul-or-not-new-insight.